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The rôle of W. Wistar Comfort in the theory of topological groups ☆

Dieter Remus

Institut für Mathematik, Universität Hannover, Welfengarten 1, D-30167 Hannover, Germany

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Abstract

The important rôle of W.W. Comfort in the area of topological groups is described.

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To Wis Comfort on the occasion of his sixtieth birthday

0. Introduction

In 1964 W.W. Comfort set foot in the area of topological groups with the paper [31] written together with K.A. Ross. Up until today he has published 45 articles on this topic. His important survey article [5] is cited in many papers written since 1984. In this paper, I will give a *selective* survey of the theorems on topological groups found by W.W. Comfort (and co-authors) in more than thirty years. The results show in an impressive manner that W.W. Comfort is one of the leading experts in that part of topological groups, where mainly *set-theoretical aspects* are considered. I am convinced that many excellent new results will follow in the future.

☆ This paper was presented by the author in the Conference of Set-Theoretic Topology in honor of W.W. Comfort held in Curaçao in June 1996. I thank A. Hager and J. van Mill for the invitation to speak about the work of my friend Wis Comfort in that conference.

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1. Totally bounded group topologies

A Hausdorff topological group G is called *totally bounded* if G is a dense subgroup of a compact group. Let $\mathcal{B}(G)$ denote the set of all totally bounded group topologies on a group G. Comfort and Ross were the first who considered this set. In 1964 they described it for Abelian groups in the following way.

Theorem 1.1 [31]. *Let G be an infinite Abelian group, and let G^* be the group of all characters of G. For each $\tau \in \mathcal{B}(G)$ let $\psi(\tau)$ be the group of all continuous characters of (G,τ). Then ψ is a bijection between $\mathcal{B}(G)$ and the set of the point-separating subgroups of G^*.*

In 1945 Markov [74] asked whether each infinite group G admits a non-discrete Hausdorff group topology. Without mentioning Markov in their work, Kertész and Szele [68] proved that every infinite Abelian group admits a non-discrete metrizable group topology. This result was improved by Comfort and Ross [31], who showed with the help of Theorem 1.1 that the assertion remains true for metrizable locally bounded group topologies. (A topological group is called *locally bounded* if it is a dense subgroup of a locally compact group.) Markov's question was negatively answered by Hesse, Ol'shanskiĭ and Shelah in 1979/1980—for further details see [12, pp. 89–91].

In 1974 Kiltinen [71] showed by using the theory of topological fields that every infinite Abelian group G has exactly $2^{2^{|G|}}$ Hausdorff group topologies. This result was independently improved by Berhanu et al. [4] and the author [80], where Theorem 1.1 is the main tool for the proof.

Theorem 1.2. *Every infinite Abelian group G admits exactly $2^{2^{|G|}}$ totally bounded group topologies.*

For an extension of Theorem 1.2 to classes of non-Abelian groups see [81]. Berhanu et al. [4] started in 1985 the investigation of the poset $\mathcal{B}(G)$ for Abelian groups G by computing cardinal invariants of $\mathcal{B}(G)$ like width, height and depth. Berarducci et al. [3] extended their results by using heavily Theorem 1.1. Comfort and the author [21] began in 1991 with the study of long chains of Hausdorff group topologies. For a group G, let $\mathcal{N}(G)$ be the set of all non-totally bounded Hausdorff group topologies for G. Let α and β be infinite cardinals. Then $C(\alpha,\beta)$ means that there is in the power set $\mathcal{P}(\alpha)$ a chain of length β. One of the main results of the mentioned paper is

Theorem 1.3 [21, Theorem 3.4]. *Let α and β be infinite cardinals. Then the following conditions are equivalent*:

(a) $C(2^\alpha,\beta)$;
(b) *some Abelian group G with $|G|=\alpha$ admits a chain $\mathcal{C}\subseteq\mathcal{B}(G)$ ($\mathcal{C}'\subseteq\mathcal{N}(G)$) with $|\mathcal{C}|=\beta$ ($|\mathcal{C}'|=\beta$)*;
(c) *every Abelian group G with $|G|=\alpha$ admits a chain $\mathcal{C}\subseteq\mathcal{B}(G)$ ($\mathcal{C}'\subseteq\mathcal{N}(G)$) with $|\mathcal{C}|=\beta$ ($|\mathcal{C}'|=\beta$).*

It is known (cf. [21, Section 1]) that $C(\omega, 2^{\omega})$ holds and $C(\kappa, \kappa^{+})$ is true for all $\kappa \geqslant \omega$, but there are models in ZFC in which $C(\kappa, 2^{\kappa})$ fails for certain κ.

Recently improvements of Theorem 1.3 were found in [26]. First some notations are introduced:

$\chi(G, \tau)$ and $w(G, \tau)$ denotes the character and weight of a topological group (G, τ), respectively. For $\alpha \geqslant \omega$ and G a group let

$$\mathcal{B}_{\alpha}(G) = \{\tau \in \mathcal{B}(G)\colon\ \chi(G, \tau) = \alpha\} \quad \text{and}$$
$$\mathcal{N}_{\alpha}(G) = \{\tau \in \mathcal{N}(G)\colon\ \chi(G, \tau) = \alpha\}.$$

It is well known that for infinite totally bounded groups (G, τ) the cardinal invariants $w(G, \tau)$ and $\chi(G, \tau)$ are the same. Let α, β and γ be infinite cardinals. Then $E(\gamma, \beta, \alpha)$ means: there is a chain $\mathcal{C} \subseteq \mathcal{P}(\gamma)$ such that $|\mathcal{C}| = \beta$ and each $C \in \mathcal{C}$ satisfies $|C| = \alpha$ (cf. [26, Section 2]).

Theorem 1.4 [26, Theorem 3.3]. *Let α, β and γ be infinite cardinals, and let G be a* (*discrete*) *maximally almost periodic group such that $|G| = |G/G'| = \gamma \geqslant \omega$, where G' is the commutator subgroup of G.*

(A) *The following conditions are equivalent*:
  (a) $C(2^{\gamma}, \beta)$;
  (b) *there is a chain $\mathcal{C} \subseteq \mathcal{B}(G)$ such that $|\mathcal{C}| = \beta$.*

(B) *Let $\log\gamma \leqslant \alpha \leqslant 2^{\gamma}$. Then either $\mathcal{B}_{\delta}(G) = \emptyset$ for all $\delta \leqslant \alpha$, or the following conditions are equivalent*:
  (a) $E(2^{\gamma}, \beta, \alpha)$;
  (b) *there is a chain $\mathcal{C} \subseteq \mathcal{B}_{\alpha}(G)$ such that $|\mathcal{C}| = \beta$.*

A result which improves, extends and subsumes Theorem 1.4 was announced and proved by Dikranjan [41,42] and co-authors [3] (for more details, see [26, Section 1]).

By using (not necessarily Hausdorff) minimally almost periodic group topologies, which are not anti-discrete, Comfort and the author could prove

Theorem 1.5 [26, Theorem 5.6]. *Let α, β and γ be infinite cardinals, and let G be a* (*discrete*) *maximally almost periodic group such that $|G| = |G/G'| = \gamma \geqslant \omega$.*

(A) *The following conditions are equivalent*:
  (a) $C(2^{\gamma}, \beta)$;
  (b) *there is a chain $\mathcal{C} \subseteq \mathcal{N}(G)$ such that $|\mathcal{C}| = \beta$.*

(B) *Let $\log\gamma \leqslant \alpha \leqslant 2^{\gamma}$. Then either $\mathcal{B}_{\delta}(G) = \emptyset$ for all $\delta \leqslant \alpha$ or* (a) *implies* (b):
  (a) $E(2^{\gamma}, \beta, \alpha)$;
  (b) *there is a chain $\mathcal{C} \subseteq \mathcal{N}_{\alpha}(G)$ such that $|\mathcal{C}| = \beta$.*

In the proof of the following theorem, the structure theory of compact, connected groups was applied heavily.

Theorem 1.6 [26, Theorem 7.1]. *Let α, β and γ be infinite cardinals, and let (G,τ) be a totally bounded group with $|G|=\gamma$ such that $w(G,\tau)=2^\gamma$ and the Weil completion of (G,τ) is connected.*

(A) *The following conditions are equivalent*:
  (a) $C(2^\gamma,\beta)$;
  (b) *there is a chain $\mathcal{C}\subseteq\mathcal{B}(G)$ such that $|\mathcal{C}|=\beta$ and $\bigcup\mathcal{C}\subseteq\tau$.*

(B) *Let $\gamma\leqslant\alpha\leqslant 2^\gamma$. Then the following conditions are equivalent*:
  (a) $E(2^\gamma,\beta,\alpha)$;
  (b) *there is a chain $\mathcal{C}\subseteq\mathcal{B}_\alpha(G)$ such that $|\mathcal{C}|=\beta$ and $\bigcup\mathcal{C}\subseteq\tau$.*

The study of chains of pseudocompact and countably compact group topologies on Abelian groups was initiated by Comfort and the author in [22, Sections 6 and 7], respectively. Recently Dikranjan [43] has proven many interesting results about chains of pseudocompact group topologies for a class of groups which contains relatively free groups and Abelian groups. For chains of pseudocompact group topologies on compact, connected groups see [26, Theorem 7.2].

Every (discrete) maximally almost periodic group G admits a finest totally bounded group topology τ_f. In 1973 Comfort and Saks [33] started the investigation of this group topology. Among other things they showed

Theorem 1.7.

(a) [33, Theorem 2.2] *Let G be an infinite Abelian group. Then (G,τ_f) is not pseudocompact.*

(b) [33, Theorems 2.5, 2.6] *Let $\{(G_i,\tau_{f_i})\colon i\in I\}$ be a family of groups, each with its finest totally bounded group topology τ_{f_i}. If $|I|<\omega$ and (G,τ) is the product of the spaces (G_i,τ_{f_i}), then τ is the finest totally bounded group topology for G. If $|I|\geqslant\omega$ and G_i is a nontrivial Abelian group for all $i\in I$, then the totally bounded group topology τ is not the finest totally bounded group topology on G.*

Theorem 1.7(a) motivated the author to study the number of non-pseudocompact, totally bounded group topologies—see [82, Kapitel 2].

In the paper [33] of Comfort and Saks it was noted that the compact group topology of the real, special orthogonal group $SO(3,\mathbb{R})$ is the finest totally bounded group topology on this group. It was Robertson who pointed out to both authors that according to a result of van der Waerden [89] $SO(3,\mathbb{R})$ admits no discontinuous homomorphism into any compact group. This leads Comfort [6, p. 195] to the following definition: A topological group (G,τ) is called a *van der Waerden group* (vdW-group) if (G,τ) is a compact group on which every homomorphism to a compact group is continuous. It can be easily seen (cf. [33, pp. 39–40]) that a compact group is a vdW-group if and only if its topology is the finest totally bounded group topology on G. Van der Waerden's result [89, p. 785] implies that every compact, connected, semi-simple Lie group is a vdW-group. (Examples of infinite, totally disconnected vdW-groups can be found in [76, p. 510 ff].) Later on Comfort and Robertson [29] discussed van der Waerden's result in the special case of $SO(3,\mathbb{R})$ at some length. In [26] vdW-groups are used to construct a class of non-Abelian groups which admit exactly one totally bounded group topology.

Theorem 1.8 [26, Theorem 3.17]. *Let $\{(G_i, \tau_i)\colon i \in I\}$ be a family of compact, algebraically simple, non-Abelian Lie groups, let τ be the product topology on $\prod_{i\in I} G_i$, and let $H = \bigoplus_{i\in I} G_i$. Then $\tau|H$ is the only totally bounded group topology on H.*

In 1960 Stewart [85, Theorem 3.1] showed the following: Let (G, τ) be a compact, connected group with a totally disconnected center. Then τ is the only compact group topology on G. Four years later Hulanicki [62, Theorem 8.10] gave an algebraic characterization of all Abelian groups which admit exactly one compact group topology. Let (G, τ) be a non-metrizable, compact, connected group with totally disconnected center. By Stewart's result τ is the only compact group topology on G, but (G, τ) is not a vdW-group, since [24, Corollary 6.7] implies that every connected vdW-group must be metrizable.

Comfort was the first who considered the important class of vdW-groups. Thus the author suggested in his talk in Curaçao the following

Definition 1.9. Let (G, τ) be a totally bounded group. If τ is the only totally bounded group topology on G, then (G, τ) is called a *Comfort group*.

Clearly every vdW-group is a Comfort group, but the converse is not true. Theorem 1.8 implies that for every infinite cardinal α there is a non-compact Comfort group G such that $|G| = \alpha$ and $w(G) = \alpha$. I will finish this section with

Problems 1.10.
(a) Characterize all non-Abelian groups which admit exactly one compact (totally bounded) group topology.
(b) Describe the class of vdW-groups (Comfort groups).

2. Pseudocompact groups

Following Hewitt's work [59], Corson [39] and Glicksberg [52] and Kister [72] introduced pseudocompactness into the context of topological groups with the following result:

If $\{X_i\colon i \in I\}$ is a family of compact spaces with product K, and if $p_i \in X_i$, then the Σ-product

$$S = \{x \in K\colon |\{i \in I\colon x_i \neq p_i\}| \leqslant \omega\}$$

is pseudocompact with $K = \beta S$. Clearly S is a topological group if each X_i is a topological group and the point p_i is the identity in X_i. Motivated by the observation that S as above is G_δ-dense in K, Comfort and Ross [32] initiated the study of pseudocompact groups. The first principal results are

Theorem 2.1 [32].

(a) *A topological group G is pseudocompact if and only if G is totally bounded and $\beta G = \overline{G}$. ($\overline{G}$ is the Weil completion of G.)*
(b) *A totally bounded group G is pseudocompact if and only if G is G_δ-dense in its Weil completion.*
(c) *If $\{G_i\colon i \in I\}$ is a family of pseudocompact groups, then $\prod_{i\in I} G_i$ is pseudocompact.*

The last assertion does not hold for products of pseudocompact spaces. Considering thus assertion Theorem 2.1(c), Arhangel'skiĭ [2, p. 15] says: "*In contrast, the following brilliant and surprising result of Comfort and Ross is specific in nature*". Theorem 2.1 was subsequently strengthened and generalized by many workers, including Hušek, Tkačenko, Trigos-Arrieta, Uspenskiĭ and de Vries (see [12, 3.6.B] for specific references). In particular, Hernández and Sanchis [57] could show in 1993 that a dense subspace A of a compact group K is pseudocompact if and only if A is G_δ-dense in K. Recently Comfort and Trigos-Arrieta [37] have characterized locally pseudocompact groups in different ways. (A topological group is said to be *locally pseudocompact* if the identity has a pseudocompact neighbourhood.) Among many interesting results they show

Theorem 2.2 [37, Theorem 3.3]. *Let G be a locally bounded group with Weil completion $\overline{G}$. Then G is locally pseudocompact if and only if $\beta G = \beta\overline{G}$.*

Another characterization is given in Theorem 7.1. Furthermore, they could improve the result of Hernández and Sanchis mentioned above by different methods.

In 1944 Halmos [55] raised the problem of classifying the algebraic structure of compact Abelian groups. It was completely solved by Hulanicki in 1957/58 (for further details see [60, §25]). In 1987 Comfort became interested in the nearly question which Abelian groups admit a pseudocompact group topology (cf. [22, p. 225]). In 1989 the author joined the investigation. In the common work the cardinal invariant $m(\alpha)$ plays an important rôle. It was introduced by Comfort and Robertson [28] in 1985: For each infinite compact group K define

$$m(K) = \min\bigl\{|G|\colon\ G \text{ is a dense pseudocompact subgroup of } K\bigr\}.$$

By [28, Theorem 2.7], $m(K)$ depends only on the weight of K. Thus one can define $m(\alpha)$ for every infinite cardinal α: Let $m(\alpha) = m(K)$ for some compact group K of weight α.

Theorem 2.3 [22, Theorem 3.3]; [23, Theorem 3.3]. *Let α and γ be infinite cardinals, let $\mathbb{Q}$ be the group of rational numbers, and let F be a non-trivial finite Abelian group. Then the following statements are equivalent*:

(a) $m(\alpha) \leqslant \gamma \leqslant 2^\alpha$;
(b) *$\bigoplus_\gamma F$ admits a pseudocompact group topology of weight α;*
(c) *$\bigoplus_\gamma \mathbb{Q}$ admits a pseudocompact group topology of weight α.*

Many other related assertions can be found in [22,23]. In [22] the following statement was introduced:

$$\text{(M)} \quad m(\alpha) = (\log \alpha)^{\omega} \quad \text{for all } \alpha \geqslant \omega.$$

(For a detailed discussion of (M), see [23, 2.4].) With the help of (M) one can give the following characterization.

Theorem 2.4 [22, Corollary 4.16]. *Let α and γ be infinite cardinals with $\alpha > \omega$, and let G be an infinite Abelian group such that $|G| = \gamma$. Assume* (M). *Then G admits a connected, pseudocompact group topology of weight α if and only if $\alpha \geqslant \log \gamma$ and the torsion-free rank $r_0(G)$ of G fulfills $r_0(G) \geqslant (\log \alpha)^{\omega}$.*

Substantial improvements and other interesting results are contained in recent, independent work of Dikranjan and Shakhmatov [48], in particular they could show (without any set-theoretical assumption) a result which contains Theorem 2.4 as a special case—see [48, 7.1]. Among other things they extended Comfort's question from above to arbitrary groups [48, Problem 0.2], and gave necessary and sufficient conditions that a free group in a variety admits a non-discrete pseudocompact group topology [48, 5.5 and 5.7]. Still open is

Problem 2.5. Classify the algebraic structure of pseudocompact Abelian groups.

In the opinion of Dikranjan and Shakhmatov the following problem seems to be the hardest obstacle to solve Problem 2.5.

Problem 2.6 [48, Problem 9.11]. Let G be a non-torsion Abelian group of torsion-free rank $r_0(G)$. $\mathrm{Ps}(r_0(G))$ means that there is a pseudocompact group of cardinality $r_0(G)$. Is $\mathrm{Ps}(r_0(G))$ a necessary condition for the existence of a pseudocompact group topology on G?

3. Pseudocompact refinements

In 1982 Comfort and Robertson [27] started the investigation of the following problem: Let (G, τ) be an infinite compact group. Does G admit a pseudocompact group topology strictly finer than τ? Among other things their work was motivated "*by a fascination with pseudocompact groups as a subject of interest in its own right*" [27, p. 173]. Their main result is

Theorem 3.1 [27, Theorem 3.4]. *Let (G, τ) be a compact Abelian group of weight $w(G)$. There is a pseudocompact group topology for G strictly finer than τ if and only if $w(G)$ is uncountable.*

By generalizing a construction initiated in [27,30] Comfort and the author [24] could prove in 1994 the following results.

Theorem 3.2 [24, Theorem 5.5]. *Let (G,τ) be a compact Abelian group such that $w(G,\tau)=\alpha>\omega$. If one of the following conditions holds, then G admits a pseudocompact group topology μ strictly finer than τ such that $w(G,\mu)=2^{|G|}$.*

(a) *The connected component C of (G,τ) satisfies $w(C)=\alpha$*;

(b) *G is a torsion group*;

(c) $\mathrm{cf}(\alpha)>\omega$.

In the general case a partial result is known.

Theorem 3.3 [24, Corollary 6.6]. *Let (G,τ) be a compact, connected group such that $w(G,\tau)=\alpha>\omega$, and let A be the component of the center of G. If $w(A)=\alpha$ or* $\mathrm{cf}(\alpha)>\omega$, *then G admits a pseudocompact group topology μ strictly finer than τ such that $w(G,\mu)=2^{|G|}$.*

With the help of Theorem 3.3 it was shown in [24] that for every compact, connected group (G,τ) of uncountable weight there is a pseudocompact group topology on G strictly finer than τ. The question whether this result holds for arbitrary compact groups is still open.

Inspired by Theorem 3.1 (cf. [1, p. 163]) Arhangel'skiĭ proved

Theorem 3.4 [1, Theorem 4]. *Let (G,τ) be a compact group such that $|G|$ is not Ulam-measurable. Then there is no countably compact group topology on G strictly finer than τ.*

This result motivated Comfort and the author to study the converse.

Theorem 3.5 [25, Theorem 5.2]. *Let m be the least Ulam-measurable cardinal. Let (G,τ) be an infinite compact group with $w(G,\tau)=\alpha$, and suppose that either* (i) *G is Abelian or* (ii) *(G,τ) is connected. Then the following conditions are equivalent*:

(a) *$\alpha\geqslant m$;*

(b) *G admits a countably compact group topology strictly finer than τ.*

Recently Uspenskiĭ [87] has shown that every (not necessarily Abelian or connected) compact group of Ulam-measurable cardinality admits a strictly finer group topology which agrees with the original one on every set of non-measurable cardinality, hence is κ-compact for every non-measurable cardinal κ. This implies that the additional hypotheses (i) and (ii) in Theorem 3.5 are unnecessary.

4. Proper dense subgroups

In 1972 Dietrich [40] posed the following question: Does every non-discrete locally compact Abelian group have a proper dense subgroup? Four years later, Rajagopalan and Subrahmanian [79] could give a negative answer. Later on Khan [69] and Kabenyuk [66, 67] characterized all non-discrete locally compact Abelian groups possessing no proper dense subgroup. In 1973 Comfort and Saks [33] proved the following result.

Theorem 4.1 [33, Corollary 1.6]. *Let α be a cardinal and let K be a compact group such that $\omega \leqslant w(K) \leqslant 2^{\alpha}$. Then K contains a dense, countably compact subgroup H such that $|H| \leqslant \alpha^{\omega}$.*

Eight years before, Itzkowitz [63,64] showed this theorem for Abelian compact groups. In 1966 Wilcox [90] could derive Theorem 4.1 for "pseudocompact" in place of "countably compact".

Definition 4.2 [30, Definition 5.1]. A pseudocompact group (G, τ) is an *extremal* (pseudocompact) group if either

(a) G has no proper, dense, pseudocompact subgroup, or

(b) there is no pseudocompact group topology μ for G such that μ is strictly finer than τ.

Comfort and Robertson made the following conjectures [30, p. 25], which are still neither proved nor disproved:

(1) (a) and (b) in Definition 4.2 are equivalent;

(2) (a), (b) fail for every pseudocompact Abelian group of uncountable weight.

The following result corroborated their conjectures.

Theorem 4.3 [30, Theorem 7.3]. *Let G be a zero-dimensional, pseudocompact Abelian group of uncountable weight. Then G is not extremal.*

In 1989 Comfort and van Mill [16] could give many sufficient conditions for the existence of a proper, dense, pseudocompact subgroup of a connected, pseudocompact Abelian group.

Theorem 4.4 [16, Theorems 4.3, 5.5, 6.1 and 7.1]. *Let G be a connected, pseudocompact Abelian group of weight $w(G) = \alpha > \omega$. If any of the following conditions holds, then G has a proper, dense* (*necessarily connected*) *pseudocompact subgroup*:

(a) $w(G) \leqslant \mathfrak{c}$;

(b) $|G| \geqslant \alpha^{\omega}$;

(c) *the torsion subgroup* $\mathrm{t}(G)$ *of G fulfills* $|\mathrm{t}(G)| > \mathfrak{c}$;

(d) *G is not divisible.*

A big progress represents the following result of Comfort et al. [9].

Theorem 4.5 [9, Theorem 1.3]. *Let G be a pseudocompact Abelian group such that $|G| > \mathfrak{c}$ or $\omega_1 \leqslant w(G) \leqslant \mathfrak{c}$. Then G has a proper, dense, pseudocompact subgroup.*

Nevertheless, it is still an open problem whether every pseudocompact group G of uncountable weight has a proper, dense, pseudocompact subgroup (what if G is Abelian? connected and Abelian?)—this is Question 501 in [7]. In particular, the following problem is open [7, Question 502]: Let G be a dense, pseudocompact subgroup of $\mathbf{T}^{\mathfrak{c}^+}$. Must G have a proper, dense, pseudocompact subgroup? (I guess that this is one of Comfort's favorite problems in the theory of topological groups.)

In [17] Comfort and van Mill discussed Abelian topological groups with and without proper dense subgroups, respectively.

Theorem 4.6.
(a) *Every non-degenerate, connected Abelian group contains a proper dense subgroup.*
(b) *Every infinite pseudocompact Abelian group contains a proper dense subgroup.*
(c) *For every strong limit cardinal α such that $\mathrm{cf}(\alpha) = \omega$ there is a totally bounded Abelian torsion group G such that $w(G) = |G| = \alpha$ and G has no proper dense subgroup.*

In 1968 Soundararajan [84] introduced the following concept. (For the Abelian case see also [86].)

Definition 4.7. A subgroup H of a topological group G is *totally dense* (in G) if $\overline{H \cap K} = K$ for every closed subgroup K of G.

This definition is closely related to the "total-minimality criterion": *A dense subgroup H of a topological group G is totally minimal if and only if G is totally minimal and $\overline{H \cap N} = N$ for every closed normal* (!) *subgroup N of G*—cf. [45, Theorem 4.3.3]. For the definition of totally minimal topological groups see Section 6.

In 1982 Comfort and Soundararajan [34] started a systematic investigation on the existence of proper, totally dense subgroups in compact groups. Let p be a prime number, and let $\mathbb{Z}(p)$ be the cyclic group of order p furnished with the discrete topology. Then it is easy to see that for every infinite cardinal α the compact group $\mathbb{Z}(p)^\alpha$ has no proper, totally dense subgroup (cf. [34, p. 71]).

Theorem 4.8 [34, Theorem 5.3]. *Let G be a compact, connected Abelian group of uncountable weight. Then G contains a proper, pseudocompact, totally dense subgroup.*

This theorem was improved by Dikranjan and Shakhmatov [46, Theorem 1.9] in 1992: *Let G be a compact Abelian group with non-metrizable component of zero. Then there exist K, an ω-bounded* (*hence countably compact*) *dense subgroup of G, and a proper, totally dense subgroup H of G such that $K \subseteq H$* (*in particular, H is pseudocompact*). (For the definition of a ω-bounded topological space see Section 7.) In the same paper they could prove the following characterization [46, Theorem 1.8] assuming Lusin's hypothesis $2^{\omega_1} = 2^{\omega}$: *A compact Abelian group G contains a proper, totally dense, pseudocompact subgroup if and only if no closed G_δ-subgroup of G is torsion.*

The paper [34] of Comfort and Soundararajan motivated Khan [70] to study the existence of proper, totally dense subgroups in locally compact Abelian groups G. Let

$\mathrm{B}(G)$ and $\mathrm{t}(G)$ denote, respectively, the subgroup of compact elements of G and the torsion subgroup of G. Khan calls G an *admissible group* if $G = \mathrm{B}(G) \neq \mathrm{t}(G)$. Among other things he shows that a locally compact Abelian group G contains a proper, totally dense subgroup if and only if G is an admissible group [70, Theorem 2]. This implies that a compact Abelian group G contains a proper, totally dense subgroup if and only if G is not a torsion group.

In 1985 Comfort studied in the joint paper [28] with Robertson the existence of small, totally dense subgroups in compact groups. (A subgroup H of a group G is called *small* if $|H| < |G|$.)

Theorem 4.9 [28, Theorem 5.6]. *Let K be a compact, connected group of uncountable weight. Then K has no small, totally dense subgroup.*

Theorem 4.10 [28, Theorem 6.3(a)]. *It is undecidable in* ZFC *whether there is a compact group with a small, totally dense, pseudocompact subgroup.*

The first of the following problems is related to Theorem 4.10.

Problems 4.11.

(a) [46, Problem 1.11] Does the characterization (given above) of compact Abelian groups having a proper, totally dense, pseudocompact subgroup hold without any additional set-theoretical assumptions beyond ZFC? Is it possible to drop “Abelian” in this characterization?

(b) [46, Problem 1.12] Let G be a compact Abelian group without closed, torsion G_δ-subgroups. Is it possible to find K, a dense, ω-bounded (or at least, countably compact) subgroup of G, and a proper, totally dense subgroup H of G such that $K \subseteq H$?

For further problems concerning the existence of proper dense subgroups the reader is referred to Section 2 of [7].

5. The Bohr topology of a LCA group

Let G be a locally compact Abelian group. Then G^+ is the group in the topology (*Bohr topology*) inherited from its Bohr compactification. In 1962 Glicksberg [53] showed that G and G^+ have the same compact subsets. For recent related results see [12, 3.6.A] and [83].

The following theorem of Comfort and Trigos [36] answers a question of van Douwen (letters to Comfort: June 1986 and May 1987), who independently gave a proof (see [49, Theorem 4.8]).

Theorem 5.1 [36, Theorem 2.2]. *Let G be a discrete Abelian group. Then G^+ is zero-dimensional.*

Recently Hernández [56] has improved this result: Let G be a locally compact Abelian group. Then $\dim G = \dim G^+$.

The special case—G is discrete—of assertion (a) in the following theorem of Comfort et al. [11] solves a long outstanding problem of van Douwen [49, Question 4.16].

Theorem 5.2 [11, Theorem 3.8]. *Let G be a locally compact Abelian group. Then the following holds*:

(a) *G is realcompact $\iff$ G^+ is realcompact $\iff$ $\kappa(G)$ is not Ulam-measurable.*

(b) *G^+ is hereditarily realcompact $\iff$ G is metrizable and $|G| \leqslant \mathfrak{c}$.*

In [49, Question 4.9] van Douwen asked whether for every subgroup H of a discrete Abelian group G the subgroup H^+ is C-embedded in G^+. The following theorem of the three authors from above contains a solution of this problem.

Theorem 5.3 [11, Theorem 5.6]. *Let G be a locally compact Abelian group, and let H be a closed subgroup of G. Then H^+ is C-embedded in G^+.*

Let Y be a Tychonoff space and $X \subseteq Y$. X is called *CM-embedded* in Y if for every completely metrizable space S the following holds: Every continuous function $f : X \to S$ extends continuously to a function $\bar{f} : Y \to S$. For discrete groups G, H^+ is even CM-embedded in G^+ (see [11, Theorem 6.3]).

In [49] van Douwen posed also the following questions: If G is a discrete Abelian group, is every countable, closed subset of G^+ a retract? For such G, is every (necessarily closed) countable subgroup of G^+ a retract? The first of these questions was answered in the negative by Gladdines [51]; the second is still open.

By the authors of [11] "*the following quite general question suggests a road for further study*".

Problem 5.4 [11, Problem 7.5]. Find an interesting class $\mathcal{C}$ of maximally almost periodic groups, containing the class of compact groups and the class of locally compact Abelian groups, such that every closed subgroup H of each $G \in \mathcal{C}$ satisfies: H^+ is C-embedded in G^+.

Glicksberg's result quoted above was generalized by Comfort et al. [38].

Theorem 5.5 [38, Theorem 2.10]. *Let G be a locally compact Abelian group and N a closed, metrizable subgroup of the Bohr compactification* bG *of G. Then every $A \subseteq G$ satisfies*:

$$A \cdot (N \cap G) \text{ is compact in } G \text{ if and only if } \{aN\colon a \in A\} \text{ is compact in } \mathrm{b}G/N.$$

In [38] examples are given to show that the equivalence can fail in the absence of the metrizable hypothesis, even when $N \cap G = \{0\}$. The authors say that a maximally almost periodic group G *strongly respects compactness* if G satisfies the property in Theorem 5.5.

Recently Galindo and Hernández [50] have given further examples of Abelian topological groups which strongly respect compactness.

6. Minimal topological groups

In 1981 Comfort wrote with Grant [10] the first survey article on minimal topological groups. A Hausdorff topological group (G, τ) is called *minimal* if there is no Hausdorff group topology on G which is strictly coarser than τ. If for every closed normal subgroup N of G the quotient G/N is minimal, then (G, τ) is called *totally minimal*. The concept of minimality was introduced independently at the beginning of the seventies by Doitchinov and Stephenson. It is a deep result of Prodanov and Stojanov that every Abelian minimal topological group is necessarily totally bounded. For more details the reader is referred to [45].

Prodanov and Stojanov [77] wrote 1983 in their survey article: "*The study of minimal group topologies began in 1971/72 and thereafter a considerable number of results concerning this subject has been obtained. Much of them are described in the survey of Comfort and Grant* [7]. *The present paper is a supplement to* [7]".

Comfort contributed also to this area. In [10] it is shown that a totally minimal (non-Abelian), countably compact group need not be compact. It follows easily from Theorem 2.1(a), the totally minimality criterion—see Section 4—and the following theorem of Comfort and Sondararajan [34] from 1982 that a totally minimal, strongly zero-dimensional, countably compact Abelian group is compact.

Theorem 6.1 [34, Theorem 6.7]. *Let G be a compact, totally disconnected Abelian group and H a totally dense, countably compact subgroup. Then $H = G$.*

Ten years later Dikranjan and Shakhmatov [46, Corollary 1.5] could show that a totally minimal, countably compact Abelian group is compact. For that, they improved Theorem 6.1 substantially: ([46, Theorem 1.4]) *No ω-bounded* (*in particular, compact*) *group contains a proper, totally dense, countably compact subgroup*. It is still an open problem whether a connected, minimal, countably compact Abelian group is necessarily compact [44, Question 5.6].

Comfort and Soundararajan [34] were the first who showed the existence of connected, totally minimal, pseudocompact Abelian groups which are not compact (cf. Theorem 4.8). Later on Dikranjan and Shakhmatov [47, Corollary 1.6] gave examples of minimal, countably compact, non-compact Abelian groups.

In 1979 Arhangel'skiĭ posed the question whether $\psi(G) = \chi(G)$ holds for every minimal topological group G. In [54] some cases are described by Comfort and Grant in which the mentioned equality holds. A negative solution of Arhangel'skiĭ's problem was independently found in 1985/86 by Guran, Pestov and Shakhmatov (for references see [12, 3.3.D]).

Recent results in the theory of minimal topological groups are described in the excellent survey article [44] of Dikranjan. The following problem seems to be the most attractive in this area.

Problem 6.2 (cf. [44, Problem 4.1]). Describe the Abelian groups that admit minimal group topologies.

7. Miscellanea

Comfort also contributed in a joint paper with van Mill [15] to the theory of free topological groups (cf. [12, p. 110]).

In the paper [13] he showed with Itzkowitz the important result: If H is a closed subgroup of a locally compact group G, then $d(H) \leqslant d(G)$.

Comfort is co-author of an article [20] with Morris, Robbie, Svetlichny and Tkačenko on suitable sets of topological groups. A subset S of a topological group G is said to be *suitable* if it has the discrete topology, is a closed subset of $G \setminus \{1\}$ and the subgroup generated by S is dense in G. By Hofmann and Morris [61] every locally compact group has a suitable set. In [20] it is shown that every metrizable group and every countable Hausdorff group has a suitable set. Examples of Hausdorff topological groups without suitable sets are also produced. Under the assumption of CH or Martin's axiom it is proven that there are examples of separable Hausdorff topological groups with no suitable set. It remains open if such examples exist in ZFC.

Comfort, Soundararajan and Trigos-Arrieta [35] introduced the class of weakly locally compact groups. A topological group G is called *weakly locally compact* if (i) G is locally bounded and (ii) every closed, G_δ-subgroup N of G satisfies: G/N is locally compact. They give the following characterization of such groups.

Theorem 7.1 [35, Theorem 3.6]. *Let G be a topological group. Then G is locally pseudocompact if and only if G is weakly locally compact.*

The motivation for proving the following theorem was a question posed in the sixties by Ross in the context of locally compact groups, later modified by others, to find conditions under which two group topologies on a group, if they have the same closed subgroups, must be equal (for more details see [35, pp. 255–256]).

Theorem 7.2 [35, Theorem 4.6]. *Let τ_1 and τ_2 be group topologies on a group G such that $\tau_1 \subseteq \tau_2$, and (G, τ_1) and (G, τ_2) have the same closed subgroups. If both (G, τ_1) and (G, τ_2) are weakly locally compact, and if either* (i) *the Weil completion of (G, τ_2) is σ-compact or* (ii) *G is Abelian, then $\tau_1 = \tau_2$.*

Comfort, Soundararajan and Trigos-Arrieta make the following conjecture:

Conjecture 7.3 [35, Conjecture 4.11]. Let τ_1 and τ_2 be weakly locally compact group topologies on a group G such that $\tau_1 \subseteq \tau_2$ and (G, τ_1) and (G, τ_2) have the same closed subgroups. Then $\tau_1 = \tau_2$.

A topological space is said to be *ω-bounded* if each of its countable subsets has compact closure. For a topological group G let $\Omega(G)$ be the set of all dense ω-bounded subgroups of G. The following theorem of Comfort and van Mill improves results of Itzkowitz and Shakhmatov [65].

Theorem 7.4 [19, Theorem 3.5]. *Let G be a compact group such that $w(G) = w(G)^{\omega}$. If G is Abelian or connected, then $|\Omega(G)| = 2^{|G|}$.*

It remains open if $|\Omega(G)| = 2^{|G|}$ holds for every compact group G of uncountable weight [19, Question 5.4].

In 1943 Hewitt [58] called a topological space X *resolvable* if there is $D \subseteq X$ such that both D and $X \setminus D$ are dense in X. Comfort and van Mill [18] gave the following definition: A group G is said to be *strongly resolvable* if for every non-discrete Hausdorff group topology τ on G there is $D \subseteq G$ such that D is resolvable in (G, τ). They have characterized all Abelian strongly resolvable groups.

Theorem 7.5 [18, Theorem 5.6]. *Let G be an Abelian group.*

(a) *If G contains no subgroup isomorphic to the group $\bigoplus_{\omega}\{0, 1\}$, then G is strongly resolvable.*

(b) *Assume* MA. *If G contains a copy of $\bigoplus_{\omega}\{0, 1\}$, then G is not strongly resolvable.*

Comfort, Gladdines and van Mill [9, Theorem 1.1] proved in 1994 that every infinite totally bounded Abelian group is resolvable. In the same year Protasov [78] showed independently a much more stronger result: Any infinite totally bounded group has a countable discrete non-closed subset. (This implies the resolvability of every infinite totally bounded group.) A topological space X is called *maximally resolvable* if it contains maximally many disjoint dense subsets (cf. [8, Notation 3.1]). Recently Malykhin and Protasov [73, Theorem 2] have proven the following: *In any infinite group G there exists a disjoint family of cardinality $|G|$ of subsets of G which are dense in any totally bounded group topology on G*. This implies that each infinite totally bounded group is maximally resolvable [73, Theorem 3].

For related and recent results on resolvable topological groups the reader should consult [8,14,75,88].

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